



Progression in Addition EYFS – Y6

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as', addend + addend = sum.

EYFS

Before addition can be introduced, children need to have a secure knowledge of number in order to begin addition. To do this, children need to become familiar with all numbers to 20 and understand what those numbers mean.

Children are then introduced to the concept of addition through practical games and activities. This is reinforced by opportunities provided in the outdoor area for the children to use addition e.g. adding together groups of building blocks, twigs etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definition. An example of this is "addition means we add two groups together / we put 2 lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5! We have got 5 altogether". Adults support children in recording their addition calculations in the written form on whiteboards and in their maths books.

Strategy	Concrete	Pictorial	Abstract
Combining two	Use other resources too e.g. eggs, shells, teddy	Children to represent the cubes using dots or	4 + 3 = 7
parts to make a	bears, cars).	crosses. They could put each part on a part whole model too.	Four is a part, 3 is a part and the whole is seven.
whole		model too.	,
(YR & Y1)	Use real life relevant examples for children.		(4)(3)
Counting on using	Using cubes or Numicon first, then alongside a	A bar model which encourages the children to	The abstract number line:
	number line.	count on, rather than count all.	What is 2 more than 4?
number lines	0 1 2 3 4 5 6 7 8 9 10	4	What is the sum of 2 and 4? What is the total of 4 and 2?4 + 2
(Y1)	4 5 6	?	4 5 6
Regrouping to make	Using ten frames and counters/cubes or using	Children to draw the ten frame and	Children to develop an understanding
10	Numicon.	counters/cubes.	of equality e.g.
(Y2-Y6)	6+5		6 + \square = 11 6 + 5 = 5 + \square 6 + 5 = \square + 4



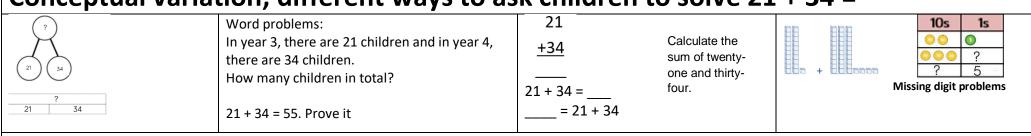


TO + Os	Using base 10. Continue to develop	Children to represent the base 10 e.g. lines for	41 + 8
	understanding of partitioning and place value.	tens and dot/crosses for ones.	
(Y2)	41 + 8	10s Is III .	41
Place value counters and bead stings can also be used.			1+8=9
beau stiligs call also be used.		4 9	40 + 9 = 49
TO + TO with	Using base 10. Continue to develop	Children to represent the base 10 in a place value	Looking for ways to make 10. (Bridging 10)
exchanging	understanding of partitioning and place	chart.	30 + 20 = 50
exeriariging	value. 36 + 25		5 + 5 = 10
()(2)	10s 1s	111	50 + 10 + 1 = 61
(Y2)			30 1 10 1 01
		11 1	36 + 25=
		6	
	6 1		1 5
			1 5
Use of place value	When there are 10 ones in the 1s column- we	Children to represent the counters in a place	Formal written method – column addition
counters to add HTO	exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.	value chart, circling when they make an exchange.	243
+ TO, HTO +		100s 10s 1s	1360
HTO etc	100s 10s 1s	00 6000 600	<u>+368</u>
HIOELC	0000 000		611
	000 0000	000 0000 0888)	1 1
	6 1 1	6	
Adding up to 5-digit	Recap above emphasising the 'exchange' process.	Recap above emphasising the 'exchange' process.	Children should be confident in using the formal written
numbers.			method of column addition. Focus on procedural variation – making links between
			calculations.





Conceptual variation; different ways to ask children to solve 21 + 34 =



Progression in Subtraction EYFS – Y6

Key language: take away, less than, the difference, subtract, minus, fewer, decrease minuend – subtrahend = difference

EYFS

Before subtraction can be introduced, children need to have a secure knowledge of number in order to begin subtraction. Children are then introduced to the concept of subtraction through practical games and activities. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge of 'less' by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary supported by age appropriate definition. An example of this is "subtraction means we take away objects from a group / we have 11 got less objects now. Equals means we find out how many we have got left. Wow! We have only got 3 left!" Adults support children in recording their subtractions in the written form on whiteboards and in their maths books.

Strategy	Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole	Ten frames, Numicon, cubes and other items such as beanbags could be used. $4-3=1$	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.	4-3 = = 4-3
		Ø Ø Ø Ø	3 ? ?





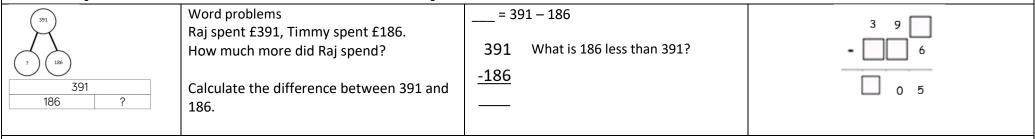
Counting back	Using number lines or number tracks Children start with 6 and count back 2. 6 - 2 = 4 1 2 3 4 5 6 7 8 9 10	Children to represent what they see pictorially e.g.	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.
Finding the difference	Using cubes, Numicon or Cuisenaire rods, other objects can also be used. Calculate the difference between 8 and 5.	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.	Find the difference between 8 and 5. 8 – 5, the difference is Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.
Making 10	Using ten frames. 14 – 5 -4 -1 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4	Children to present the ten frame pictorially and discuss what they did to make 10.	Children to show how they can make 10 by partitioning the subtrahend.
Using partitioning.	Using base 10 or place value counters 48-7 10s 1s 10s 1s 4 1	Children to represent the base 10 pictorially.	Children to write out their mental strategy. $48-7=$ $40-0=40$ $8-7=1$ $40+1=41$
Using partitioning with an exchange – leading to column subtraction.	Using base 10 or PV counters and having to exchange. 41 – 26	Represent the base 10 pictorially, remembering to show the exchange.	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.





	10s 1s 10s 1s 1s 10s 1s 1s 5	10s 1s 1+10 ::::::	- 1 1 5
0 1	Recap above emphasising the 'exchange' process.	Recap above emphasising the 'exchange' process.	Children should be confident in using the formal written method of column subtraction. Focus on conceptual variation and procedural variation.

Conceptual variation; different ways to ask children to solve 391 - 186



Progression in Multiplication EYFS – Y6

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups, exchange. Multiplicand x multiplier = product

EYFS

By the end of EYFS, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically add two equal groups together to find out the 'doubles' answer.





Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.	Draw pictures to show how to double a number. Double 4 is 8	16 10 6 10 x2 20 12 Partition a number and then double each part before recombining it back together.
Repeated grouping / Repeated addition	'There are 3 equal groups, with 4 in each.' $4+4+4=12$	Children to represent the practical resources in a picture and use a bar model. There are 3 plates, each plate has 2 apples. How many apples are there all together?	Write addition calculations to describe pictures and objects. 2 + 2 + 2 = 6
Counting in multiples		Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30 Start from different numbers to count in multiplies, not just zero.





	Count in multiples supported by concrete objects in equal groups.		8, 10, 12, 14 etc.
Number lines to show repeated groups	Numicon Use a ruler to show a number line. Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 4 + 4 + 4 = 12 5 5 5 5 5 12 13 14 15 5 + 5 + 5 = 15	Abstract number line showing three jumps of four. $3 \times 4 = 12$
Use arrays to illustrate commutativity	Counters and other objects can also be used. $2 \times 5 = 5 \times 2$ 2 lots of 5 $5 lots of 2$	Children to represent the arrays pictorially.	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply	Using Numicon, base 10 (Dienes) or Cuisenaire rods. 4 × 15 =	Children to represent the concrete manipulatives pictorially. Os Is Clearly show the 'exchange' of the 20 ones for 1 ten.	Children to be encouraged to show the steps they have taken. 4 \times 15 10 5 A number line can also be used. 10 \times 4 = 40 5 \times 4 = 20 40 + 20 = 60





Grid Method	Show the link with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method 4 rows of 13	They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. 30 5 7 210 35 210 + 35 = 245
Formal column method no exchanging. 1-digit x 2-digit (Y3) 1-digit x 3-digit (Y4) 1-digit x 4-digit (Y5)	Place value counters can be used to represent (base 10 can also be used.) 3 × 23 10s 1s 6 9	Children to represent the counters pictorially. 10s Is 00 000 00 000 6 9	Children to record what it is they are doing to show understanding. $3 \times 23 \qquad 3 \times 20 = 60$
Formal column method with exchanging. 1-digit x 2-digit (Y3) 1-digit x 3-digit (Y4) 1-digit x 4-digit (Y5)	Place value counters or Base 10 can be used to represent 6 x 23 100s 10s 1s 100s 1s 100s 1s 100s 1s 100s 1s 100s 1s 10s	Children to represent the counters/base 10, pictorially e.g. the image below. Children to be exposed to the formal written method alongside the pictorial.	Formal written method $6 \times 23 = 23$ $\frac{\times 6}{138}$ $\frac{1}{1}$





Formal column method (Long multiplication)	Represent using Base 10 and Place value counters using the grid method.	Moving forward, multiply by a 2 digit number showing the different rows within the grid	Formal written method of long multiplication.	
2-digit x 2-digit.	× 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	method. 10 8 10 80 3 30 24	2 3 x 1 4 9 2 (23 x 4) 2 3 0 (23 x 10)	
Formal written method	When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract	When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract The grid method can be used as a pictorial representation.	Formal written method of long multiplication.	
Multiplying decimals up to 2 decimal places by a single digit.	Place Value counters can be used if necessary.	Place Value counters can be drawn if necessary.	Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer. 3 · 1 9 × 8 2 5 · 5 2	

Conceptual variation; different ways to ask children to solve 6 × 23

Bar model	Word problems	Find the product of 6 and 23	What is the calculation?
23 23 23 23 23 23 23 23 23 23 23 23 24x = 20 20 4	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that 6 x 23= 138	6 × 23 = 6 23 = 6 × 23 × 6 =	What is the product? 100s 10s 1s





Progression in Division EYFS – Y6

Key language: share, group, divide, divided by, half, repeated subtraction **Dividend** ÷ **divisor** = **quotient**

EYFS

By the end of EYFS, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.

Strategy	Concrete	Pictorial	Abstract
Halving	Using a range of objects.	Represent the sharing pictorially.	6 ÷ 2 = 3 Children should also be encouraged to use their 2 times table facts.
Sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8+2=4 Children use bar modelling to show and support understanding.	12 ÷ 3 = 4





Grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping. **Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. **Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. **Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Repeated subtraction	Using Cuisenaire rods above a ruler. 6 ÷ 2	Children to represent repeated subtraction pictorially.	Abstract number line to represent the equal groups that have been subtracted.
2d ÷ 1d with remainders	Using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.	Children to represent the lollipop sticks pictorially.	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over'





	There are 3 whole squares, with 1 left over.	There are 3 whole squares, with 1 left over.	5 9 13
Sharing using place value counters.	42 ÷ 3 = 14 10s 1s 10s 1s 10s 1s 10s 1s 10s 1s 10s 1s	Children to represent the place value counters pictorially.	Children to be able to make sense of the place value counters and write calculations to show the process. $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$
Short division	Using place value counters to group. 615 ÷ 5 100	Represent the place value counters pictorially.	Children to the calculation using the short division scaffold. (Division bracket) $\frac{123}{5^{1}1^{5}}$





Long division

Using place value counters 2544 ÷ 12

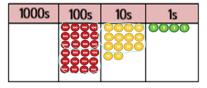
1000s	100s	10s	1s
•	0000		0000

We can't group 2 thousands into groups of 12 so will exchange them.

1000s	100s	10s	1s
	0000 0000 0000 0000	0000	0000
	9000		

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r}
 02 \\
 \hline
 12 2544 \\
 \underline{24} \\
 1
 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

	021	
12	2544	
	24	
	14	
	12	
	2	

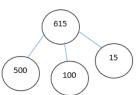
0 2 1

1000s	100s	10s	1s
	0000	0000	8888
	8000		8888

After exchanging the 2 tens, we 12 $\boxed{\frac{0212}{2544}}$ have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder. $\boxed{\frac{14}{12}}$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

5 615

615 ÷ 5 =

___ = 615 ÷ 5

What is the calculation? What is the quotient?

100s	10s	1s
888		00000 00000 00000